Abstract

Norman Robert Campbell was a scholar renowned internationally for his rigorous analysis of the foundations of physical measurement. In 1933 and 1935 he anticipated ideas that are used today for the classification of sensory scales and for nonmetric scaling.

Long ago physicists invented important psychophysical ideas, some of which attracted immediate attention while others went unnoticed from the time they were first published. For example, the logarithmic law (Fechner, 1860), the power law (Plateau, 1872), and the method of magnitude estimation (Richardson, 1928) became soon a topic of scientific interest. On the other hand, Norman Robert Campbell proposed important psychophysical ideas that never became the subject of scientific discussion.

Campbell (1919, 1921, 1928) is known internationally for his important philosophical discussion of the foundations of physical science. In psychophysics he is cited often for his philosophical analysis of physical measurement. For example, his fame is shown by the fact that he is essentially the only author considered in Stevens’s (1946) classic article On the theory of scales of measurement.

Campbell authored two articles on sensory measurement where he anticipated modern ideas for the classification of sensory scales; he also anticipated nonmetric scaling to circumvent the dubious assumption that humans correctly report sensory ratios (Campbell 1933; Campbell & Marris, 1935).

Measurement scales

Stevens (1946) proposed four classes of scales: nominal, ordinal, interval, and ratio. These classes may be briefly defined as follows. When magnitudes can be identified but information about their order is lacking, one has nominal scales. In this case, numerals are used only as names. If one can order magnitudes but cannot determine the differences between magnitudes, one has ordinal scales. If one knows the differences but cannot determine the ratios between magnitudes, one has interval scales. Finally, if one knows the ratios between magnitudes, one has ratio scales.

Campbell (1933, p. 566) asserted that “one purpose at least of measurement is to distinguish things. The numerals assigned in measurement are, in part at least, names. In this respect a method of measurement would be ideal if it always led us to assign exactly the same numerals, the same name, to the same object, and always to assign different numerals to objects that are in any way distinguishable in the property we are considering.” Scales that fulfil this “purpose” define the nominal class of scales proposed by Stevens.

Campbell continued (1933, p. 567): “What is the criterion by which we are to determine whether two methods of measurement measure the same thing? This question has often been asked; many people seem to find a strange difficulty in answering it. To me the answer seems perfectly clear, and to depend on the conception of order; this conception must be
fundamental in measurement, because the obvious reason for giving to numerals a unique position as names is their possession of a very definite and complete kind of order. My answer is that two methods measure the same thing, if the order of the numerals assigned by one to the members of a group is always the same as the order of the numerals assigned by the other to those same members.” Scales that fulfil this “criterion” define the ordinal class of scales proposed by Stevens.

Campbell & Marris (1935) noted that: “Given any pair of sounds \( U, V \) of the same frequency and any other sound \( X \), an observer can find a sound \( Y \) whose loudness is related to the loudness of \( X \) as the loudness of \( V \) is related to the loudness of \( U \) (p. 158).” The kind of measurement of loudness that one obtains depends on the arithmetical relation that is assumed to be equivalent to the quantitative relation between the loudnesses in these pairs of sounds. With \( u \) and \( v \) being the loudnesses of \( U \) and \( V \), respectively, if one assumes that the quantitative relation between \( U \) and \( V \) is equivalent to \( u - v \) then this “means that the loudness of equally related sounds are to be represented by numerals with a constant arithmetical difference. For example, the sound of loudness 100 must be related to that of loudness 90 as the sound of loudness 60 is to that of loudness 50 (p. 161)” Scales that fulfil this “assumption” define the interval class of scales proposed by Stevens.

Campbell & Marris (1935, p. 162) asserted that “another possible assumption…is that…\( v / u \) so that the loudnesses of equally related sounds are to be represented by numerals in a constant ratio.” Scales that fulfil this “assumption” define the ratio class of scales proposed by Stevens.

Immediately after Campbell & Marris’s (1935) article The measurement of loudness had been published, Stevens (1936) wrote the article A scale for the measurement of a psychological magnitude: loudness in which he restated approximately the same concepts of Campbell & Marris: “We devise scales for the purpose of facilitating the description of natural phenomena in terms of functional relationships expressed, if possible, by symbols of conventional mathematics. Consequently, it is desirable to assign numbers in each scale which not only denote the order within the scale (for which the letters of the alphabet would serve well enough), but also designate the relative magnitudes of the phenomena to which the scale is applied. When this is done, the scale numbers can be manipulated in accordance with arithmetical laws in order to determine additional relationships such as the sum of two magnitudes, the relative separation of two pairs of magnitudes, etc. (p. 405)”

Surprisingly, Stevens (1936, 1946, 1951, 1968, 1975) never mentioned Campbell’s 1933 and 1935 articles on sensory measurement. In the book Hearing, which according to Teghtsoonian (2001) “has been ranked by some scholars as second only to the work of Helmholtz,” Stevens and Davis (1938) did not cite Campbell & Marris’s (1935) article, notwithstanding that its title, The measurement of loudness, was highly pertinent to the subject of the book. We ignore whether Stevens and Davis (1938) neglected this article or did not know about it. We can only note that they neglected or did not know about other relevant articles on hearing as well. For example one of these neglected articles was by Kingsbury (1927) who reported—before Fletcher and Munson (1933)—the effect of sound frequency on loudness.

Nonmetric scaling

The fundamental disagreement between Campbell and Stevens was the following. Campbell (1933) argued extensively that it may not be true that observers have the ability to report correctly the ratio between two sensory intensities. Instead, Stevens (1936) argued that “a subjective scale is a scale of response, and the response of the observer who says ‘this is half as loud as that’ is one which, for the purpose of erecting a subjective scale, can be accepted at its face
value (p. 407).” There is now abundant evidence showing that Campbell was right and Stevens wrong. The following are two important examples of this evidence.

Consider the ratios $v_1 = \psi_a / \psi_b$, $v_2 = \psi_b / \psi_c$, and $v_3 = \psi_a / \psi_c$ and their numerical estimates $n_1$, $n_2$, and $n_3$, respectively, where $\psi_a$, $\psi_b$, and $\psi_c$ are sensory intensities. Since $v_1 v_2 = v_3$, the equality $n_1 n_2 = n_3$ occurs only when $n_i = v_i$, with $i = 1, 2, 3$. In disagreement with Stevens’s contention that $n_i$ is a measure at face value of the true ratio $v_i$, it turns out empirically that $n_1 n_2 \neq n_3$ and thus that $n_i \neq v_i$ (Fagot & Stewart, 1968).

Consider the sensory ratios $v_1 = \psi_a / \psi_b$, $v_2 = \psi_b / \psi_c$, and $v_3 = \psi_a / \psi_c$ and their numerical estimates $n_1$, $n_2$, and $n_3$, respectively, where $\psi_a$, $\psi_b$, and $\psi_c$ are sensory intensities. The equality $v_1 v_2 = v_3$ occurs when $v_2 = v_3$. Consequently, when $n_1 n_2 = n_3$, the equality $\psi_2 = \psi_3$ occurs when $n_2 = v_2$. In disagreement with Stevens, it turns out empirically that $\psi_2 \neq \psi_3$ when $n_1 n_2 = n_3$ (Ellermeier & Faulhammer, 2000; Zimmer, 2005).

To circumvent the ignorance of whether observers can report sensory ratios correctly—or whether they can report sensory differences correctly—Campbell (1933) proposed a way of measuring sensations based only on the assumption that observers can order sensory differences. In the section of his article titled Measurement by ordering differences, he did this in the following terms: “Almost everyone will agree, not only that a buttercup is yellower than milk and milk than snow, but also that the difference between a buttercup and milk is greater than the difference between milk and snow. Now it can easily be shown that if we could order in this way all the differences between sensations, that is to say not only first differences, but also second differences, third differences, and so on indefinitely, then a process of measurement would be possible by means of which we could assign numerals quite uniquely. (Perhaps I had better explain, for the benefit of the possible psychologist reader, that second differences are differences of first differences and so on.) A simple example will show what I mean. If we have 5 hues, $A, B, C, D, E$, and a choice of the numerals 0 to 10 to assign to them, and if we know that the hues stand in that order, then we can assign the numerals in many ways, so that the order of the numerals agrees with the order of the hues. But if we know also that the differences between the hues stand in the order $A-B, B-C, C-D, D-E$, then we have only one choice; the assignment must be $A\ 10, B\ 6, C\ 3, D\ 1, E\ 0$. If we knew the order of the second differences, the choice would be uniquely determined, even if we had more numerals at our disposal. Here is a system of measurement theoretically possible; the algebra of it is simple, but need not be elaborated. For as a matter of fact, we can rarely, if ever, order any differences higher than the first or second (p. 571).”

Campbell’s concept of the recovery of metric information from order information resurfaced about twenty years later when nonmetric scaling was first developed (Abelson & Tukey, 1959, 1963; Coombs, 1950, 1964; Fagot, 1959; Parker & Schneider, 1974; Schneider, 1980; Schneider, Parker, & Stein, 1974; Shepard, 1966; Siegel, 1956; Hubert & Arabie, 1986). None of the authors that contributed to this technique showed signs of being aware of Campbell’s 1933 article. The following is Schneider’s (1982, pp. 323-324) excellent description of the technique: “Suppose we have a set of points on a line segment whose coordinate values ($x_i$) are known. For $n$ such points there are $n(n-1)/2$ distinct interpoint distances. These interpoint distances can be ranked from largest to smallest. Shepard (1966) has shown that the rank order of these distances is sufficient to determine a set of projection values $P_i$ on a second line segment such that $P_i = a x_i + b$. That is to say, provided that the number of points is sufficiently large ($n \geq 10$), the rank order of interpoint distances can be used to determine projection values along a line segment that are, for all practical purposes, unique up to addition and multiplication by a constant...The reason why we need only the ordinal properties of the distances is not always intuitively obvious to those who are first introduced to this fact...Suppose we start off with four points ($a, b, c, x$) and suppose we know that the distance between $a$ and $c$, $d(a, c)$, is the largest interpoint distance. We can arbitrarily locate $a$
and c any place along the line segment. We then know that all the other points (namely, b and x) must lie between a and c; otherwise \(d(a, c)\) could not be the largest distance. Suppose we also know that \(d(a, x) > d(c, x)\). If this is true, then x must be located somewhere in the right-hand half of the line segment between a and c. Suppose also that \(d(a, b) > d(b, c)\). By the same logic b must be located in the right half of the same segment. Note, however, that we do not know whether b is to the left or right of x. If we also determine that \(d(a, x) > d(a, b)\), we know that x must be to the right of b. Finally, suppose that \(d(b, x) > d(c, x)\). This can only be true if the location of x is restricted to the rightmost quarter of the line segment. For suppose we tried to put x anywhere else. Then, \(d(b, x) > d(c, x)\) would mean that b would have to be located in the left-hand half of the line segment, something that is forbidden by \(d(a, b) > d(b, c)\). The inequalities just mentioned are sufficient to restrict the location of x to \(\frac{1}{4}\) of the line segment. Suppose we had more points and more inequalities. Then, of course, the portion of the line segment over which x could range (its neighborhood) would be decreased. In general, as the number of inequalities increases, the neighborhood over which x can range becomes progressively smaller. It follows that with a sufficiently large number of inequalities the location of x would be restricted, for all practical purposes, to a single point on the line segment between a and c. Shepard (1966) has shown that this point is effectively reached for \(n \geq 10\). Because the choice of the coordinate values for points a and c was arbitrary, it follows that the rank order of interpoint distances is sufficient to recover the location of the points on a line segment up to addition and multiplication by a constant.”

Campbell’s 1933 and 1935 articles have been neglected or have been unknown by virtually every psychophysicist. No mention of these articles can be found in subsequent important books and reviews of psychophysical scaling such as those of Anderson (1982), Baird (1997), Baird & Noma (1978), Bolanowski & Gescheider (1991), Carterette & Friedman (1974), Falmagne (1985, 1986), Gescheider (1997), Guilford (1954), Gulliksen & Messick (1960), Krueger (1989), Marks (1974), Marks & Algom (1991), Michell (1997), Moskowitz, Scharf, & Stevens (1974), Murray (1993), Newman (1974), Reese (1943), Savage & Ehrlich (1992), Torgeson (1958), and Warren (1981) among others. Campbell's articles are certainly worthy of the attention of psychophysicists, as much today as seventy years ago.

References


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