IS SELF-ESTIMATED LINEAR LENGTH LINEAR?

Sergio Cesare Masin
Department of General Psychology, University of Padua, I-35141 Padova, Italy
scm@unipd.it

Abstract

There is wide evidence that magnitude estimates of apparent length are essentially linearly related to physical length, but it is yet unknown whether these self-estimates are linear, that is, whether they are linearly related to apparent length. Gage in 1934 and Pfanzagl in 1959 devised two tests that would help resolve this problem. In the study presented here, these tests were used to select subjects producing equidifferent apparent lengths. Mean magnitude estimates and mean ratings of equidifferent apparent lengths were found to be linearly related to these same lengths, thus showing that self-estimates of apparent length were linear.

In this paper we consider the length of lines presented frontally, hereafter called length. Absolute magnitude estimates of apparent length are related to physical length almost linearly, with exponents ranging from 0.95 to 0.98 (Verrillo, 1982, 1983). The exponents obtained by conventional magnitude estimation are similar in size although they depend on the standard and modulus (Pitz, 1965; Wong, 1963). As shown hereinafter, in spite of the well-established relation between magnitude estimates of apparent length and physical length, the relation of magnitude estimates of apparent length to apparent length is indeterminate.

Attneave (1962) had distinguished among the ordinary psychophysical function with exponent $n$ (relating magnitude estimates of apparent length to physical length), the sensory function with exponent $k$ (relating apparent length to physical length), and the response function with exponent $m$ (relating magnitude estimates of apparent length to apparent length). These functions imply that

$$n = k \cdot m \quad (1)$$

(Curtis, Attneave, & Harrington, 1968).

Baird, Kreindler, & Jones (1971) have shown that the way subjects select numbers in magnitude estimation affects $n$ indirectly by affecting $m$. They used lines with length varying from 2 to 64 cm as stimuli. Eight groups of subjects magnitude-estimated the apparent length of these lines in two sessions. In the first session, the shortest line was used as the standard with modulus 1. The resulting mean $n$s of the eight groups ranged narrowly from 0.86 to 1.01. In the second session the standard and modulus were the same as before but now the same eight groups were additionally told which number to use as a magnitude estimate for the longest line. This prescribed number differed for each group. Depending on the prescribed number, the resultant mean $n$s of the groups ranged widely from 0.31 to 2.55. Since the stimulus lines were the same for all groups in both sessions, $k$ was the same in each group and session. Consequently, the prescribed number determined a change in $n$ because it determined a change in $m$ and not in $k$. Since it is virtually impossible to know beforehand how the subjects select their numbers in magnitude estimation, Baird et al.’s (1971) results show that it is indeterminate whether $m = 1$, that it, whether the response function is linear.


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One could test the linearity of the response function by nonmetric scaling. This technique recovers interval measures of apparent length from subjects' judgments of the order of differences in length between pairs of lines, without using the concept of the response function. Since it is found empirically that n is almost 1, one concludes that m is almost 1 if one finds that the recovered interval measures of apparent length are linearly related to physical length (k = 1). However, nonmetric scaling has yielded a k for apparent length ranging from 0.5 to 1 depending on the computational procedure, the experimental design, and the quantitative relation between line lengths used to order the line pairs (Fagot, 1982; Markley, Ayers, & Rule, 1969; Parker, Schneider, & Kanow, 1975; Schneider & Bissett, 1988; Young, 1970). Nonmetric scaling cannot therefore reliably determine whether the response function is linear.

In the present study the possible linearity of the response function was tested as follows. Subjects were first run on a bisection task. Given the apparent lengths \( \Psi_A \) and \( \Psi_C \), with \( \Psi_A < \Psi_C \), a bisection is the selection of the apparent length \( \Psi_B \) such that the difference between \( \Psi_A \) and \( \Psi_B \) equals that between \( \Psi_B \) and \( \Psi_C \) (Plateau, 1872). It is defined by the model

\[
\Psi_B = w \cdot \Psi_A + (1 - w) \cdot \Psi_C
\]  

(2)

with \( w \) a coefficient between 0 and 1 (Pfanzagl, 1959).

Let \( \Psi_1 \) with \( \Psi_1 < \Psi_1_{44} \) and \( i \) an integer from 1 to 5, be apparent lengths. Starting with \( \Psi_1 \) and \( \Psi_5 \), subjects made the bisections defined by the equations

\[
\Psi_2 = w \cdot \Psi_1 + (1 - w) \cdot \Psi_5
\]  

(3)

\[
\Psi_3 = w \cdot \Psi_2 + (1 - w) \cdot \Psi_5
\]  

(4)

\[
\Psi_4 = w \cdot \Psi_3 + (1 - w) \cdot \Psi_5
\]  

(5)

\[
\Psi_5 = w \cdot \Psi_4 + (1 - w) \cdot \Psi_5
\]  

(6)

Simple algebra shows that Equations 3-6 imply that \( \Psi_3 = \Psi_3 \) when \( w = 0.5 \) (Gage, 1934). The test of whether \( \Psi_3 = \Psi_5 \), called Gage's test, was used to select the subjects with \( w = 0.5 \). The subjects with \( w = 0.5 \) as found by Gage's test were subsequently used for the following test.

In Equations 3-6, Gage's test does not guarantee that \( w = 0.5 \) since different combinations of different values of \( w \) could cause \( \Psi_3 = \Psi_5 \). To determine whether \( w \) is equal to 0.5 in each of Equations 3-6, one needs to apply Pfanzagl's (1959) test, which is as follows.

Consider \( \Psi_1, \Psi_2, \Psi_3, \) and \( \Psi_4 \) (Equations 3-6). Subjects made the bisections defined by the equations

\[
\Psi_{12} = w \cdot \Psi_1 + (1 - w) \cdot \Psi_2
\]  

(7)

\[
\Psi_{34} = w \cdot \Psi_3 + (1 - w) \cdot \Psi_4
\]  

(8)

\[
\Psi_{14} = w \cdot \Psi_1 + (1 - w) \cdot \Psi_4
\]  

(9)

\[
\Psi_{32} = w \cdot \Psi_3 + (1 - w) \cdot \Psi_2
\]  

(10)

and the bisections defined by the equations

\[
\Psi_{1234} = w \cdot \Psi_{12} + (1 - w) \cdot \Psi_{34}
\]  

(11)

\[
\Psi_{1432} = w \cdot \Psi_{14} + (1 - w) \cdot \Psi_{32}
\]  

(12)

For any constant \( w \), simple algebra leads to the equality

\[
\Psi_{1234} = \Psi_{1432}.
\]  

(13)

This equality applies only if \( w \) is constant in Equations 7-12. Consequently, Pfanzagl's test, that is, the test of whether \( \Psi_{1234} = \Psi_{1432} \), allowed to select the subjects with constant \( w \).

Thus, in each bisection, \( w = 0.5 \) in the subjects who passed Gage's test and Pfanzagl's test. Since the apparent lengths \( \Psi_1, \Psi_2, \Psi_3, \Psi_4, \) and \( \Psi_5 \) produced by these subjects were equidifferent, the arbitrary equidifferent numbers 1, 2, 3, 4, and 5 were chosen as equal-interval bisection measures of the apparent lengths \( \Psi_1, \Psi_2, \Psi_3, \Psi_4, \) and \( \Psi_5 \), respectively.

The test of the linearity of the response function was made on the subjects who passed Gage's test and Pfanzagl's test. These subjects were asked to magnitude-estimate \( \Psi_1, \Psi_2, \Psi_3, \Psi_4, \) and \( \Psi_5 \) in one session and to rate these same apparent lengths in another session. It was predicted that the bisection measures (that is, 1, 2, 3, 4, and 5) were linearly related to the corresponding mean magnitude estimates and mean ratings, but only in the event that the respective response functions were linear (\( m = 1 \)).

**Method and Procedure**

Thirty-one undergraduate university students served as paid subjects. The stimuli were 1-mm wide black horizontal lines presented for 0.5 sec concentrically with a 103 × 53 cm rectangular white area of a frontal display screen (NEC PlasmaSync 50MP2 plasma monitor driven by an Intel Pentium 4 computer). The viewing distance was 1.7 m.

Three test stimuli, Stimuli A, B, and C, were presented successively for bisection in this order, with interstimulus intervals of 1 sec. After 1 sec from the offset of Stimulus C, a variable stimulus equal to Stimulus B (hereafter called the variable) was presented in the position of Stimulus B. Two keys juxtaposed horizontally and two keys juxtaposed vertically were used to vary the variable. The variable was decreased when the left or bottom key was kept pressed and was increased when the right or top key were kept pressed. The variable varied in steps of 2.1 mm when the horizontally juxtaposed keys were kept pressed and varied in steps of 5.2 mm when the vertically juxtaposed keys were kept pressed.

The subjects made bisections selecting a physical length for Stimulus B as follows. For each bisection, the test stimuli were initially presented with the physical length for Stimulus B equal to that of the terminal Stimulus A or C. Participants were asked to vary the physical length for Stimulus A was equal to that between the variable and the terminal Stimulus C. The resulting physical length of the variable was assigned to Stimulus B. So that the participant judged whether this new physical length of Stimulus B produced a satisfactory bisection, the test stimuli were presented again successively when the participant pressed a key. When the participant was not satisfied that Stimulus B produced the bisection, the participant pressed again the variable, finding another physical length for Stimulus B that more precisely produced the bisection. The test stimuli were then presented again when the participant pressed a key, with Stimulus B having this other physical length. The participant repeated this process as many times as needed until the participant was satisfied that Stimulus B produced the bisection. Participants were allowed to arrive at their bisections by bracketing.

**Gage's test.** Initially, each subject made the bisections defined by Equations 3-6. With the time order of the terminal Stimuli A and C and the initial values of Stimulus B counterbalanced, each participant made 8 times consecutively the bisection defined by Equation 3 (the mean of the resulting bisection points produced the apparent length \( \Psi_3 \)), then made 4 times in random order each of the bisections defined by Equations 4 and 5 (the mean of the resulting bisection points produced the apparent lengths \( \Psi_3 \) and \( \Psi_4 \), respectively), and finally made 4 times consecutively the bisection defined by Equation 6 (the mean of the resulting bisection points produced the apparent length \( \Psi_3 \)). The physical lengths of the terminal stimuli for the bisection defined by Equation 3 were 2 and 68 cm.
Pfanzagl's test. When Gage's test was terminated, each subject made the bisections defined by Equations 7-10. For these bisections, the length for Stimuli A and B was 2 or 68 cm (corresponding to $\Psi_1$ and $\Psi_2$, respectively) or the mean bisection points corresponding to $\Psi_3$ and $\Psi_5$ (obtained with the bisections defined by Equations 4 and 3, respectively). With the same counterbalancing conditions as before, each participant made 4 times in random order each of the bisections defined by Equations 7-10 (the mean of the resulting bisection points produced the apparent lengths $\Psi_{15}$, $\Psi_{16}$, $\Psi_{17}$, and $\Psi_{18}$, respectively), and finally made 4 times in random order each of the bisection defined by Equations 11 and 12 (the mean of the resulting bisection points produced the apparent lengths $\Psi_{1235}$ and $\Psi_{1325}$, respectively).

Absolute magnitude estimation. When Pfanzagl's test was terminated, subjects were asked to magnitude-estimate the apparent lengths $\Psi_1$, $\Psi_2$, $\Psi_3$, and $\Psi_5$. Only Stimulus C was presented. The presentation conditions of this stimulus were as before. The physical length for this stimulus was the physical length corresponding to $\Psi_1$, $\Psi_2$, $\Psi_3$, $\Psi_4$, or $\Psi_5$. The series of these five lengths were presented four times consecutively each time in random order. Each subject was read the instructions for absolute magnitude estimation of length reported in Verrillo (1983) and was then asked to magnitude-estimate the apparent length of Stimulus C.

Fig. 1. Left: mean number assigned to apparent length–mean magnitude estimate (○) or mean rating (●) as a function of the bisection measure of apparent length. Right: mean absolute magnitude estimate (AME) of apparent length plotted against mean rating of apparent length.

Rating. When the absolute magnitude estimation was terminated, subjects were asked to rate the apparent lengths $\Psi_1$, $\Psi_2$, $\Psi_3$, $\Psi_4$, and $\Psi_5$. Stimuli A, B, and C were presented with the same presentation conditions as before. Now, Stimuli A and B were used as anchor stimuli. The lengths of these stimuli were 0.5 and 93 cm, respectively. Participants were told that Stimuli A and B were reference stimuli having length 1 and 100, respectively. The series of five lengths of Stimulus C were presented four times consecutively each time in random order. Subjects were asked to rate the length of Stimulus C considering that 1 was the rating of the shorter anchor length and 100 that of the longer anchor length.

Results and Discussion

For each subject, t-tests with significance level of 0.05 were applied to determine whether $\Psi_3$ was statistically equal to $\Psi_5$ and whether $\Psi_{1235}$ was statistically equal to $\Psi_{1325}$, that is, to determine whether the subject passed Gage's test and Pfanzagl's test, respectively. Only 3 subjects failed to pass Pfanzagl's test ($\Psi_{1235} \neq \Psi_{1325}$)—one passed Gage's test and the other two did not. Of the remaining 28 subjects, 10 passed Gage's test ($\Psi_3 = \Psi_5$) while the other 18 did not. For each of these 18 subjects, $\Psi_3 < \Psi_5$ implying that $w > 0.5$.

Considering that $w = 0.5$ for the 10 subjects who passed both Gage's test and Pfanzagl's test, for these subjects the equidifferent bisection measures 1, 2, 3, 4, and 5 were assigned to the equidifferent apparent lengths $\Psi_1$, $\Psi_2$, $\Psi_3$, $\Psi_4$, and $\Psi_5$, respectively. For the 18 subjects that passed Pfanzagl's test but did not pass Gage's test no bisection measure could be derived since the value of $w$ in Equations 3-6 was unknown.

Figure 1 shows the results for the 10 subjects who passed both Gage's test and Pfanzagl's test. The left diagram shows the mean absolute magnitude estimates (open circles) and mean ratings (filled circles) of apparent length plotted against the bisection measures of apparent length. One can see that bisection measures were linearly related to mean magnitude estimates and mean ratings. Both linear trends were significant $F(1,9) = 8.8, p < 0.05$, and $F(1,9) = 623, p < 0.001$, respectively with all higher order trends not significant. Therefore, the response function was essentially linear for both magnitude estimates and ratings. Accordingly, as illustrated in the right diagram in Figure 1, the relation between mean absolute magnitude estimates and mean ratings was also essentially linear.

In the present study, the majority (64%) of the subjects who passed Pfanzagl's test failed to pass Gage's test. The reason for these failures could be the following. It has been found that subjects behave as if they were able to judge both ratios and differences between apparent lengths, with many subjects judging ratios of apparent length when they are asked to judge differences in apparent length (Parker, Schneider, & Kanow, 1975; Schneider & Bissell, 1988). In agreement with these findings, in the present study the subjects who failed to pass Gage's test most probably judged ratios rather than differences in apparent length when they made their bisections. If so, one predicts that bisections with failure to pass Gage's test should be less frequent for sensory attributes, such as loudness, brightness or heaviness, with respect to which most of the subjects behave as if they were judging sensory differences when they are asked to judge sensory ratios (Birnbaum & Elmanian, 1977; Masin, 2007; Mellers, Davis, & Birnbaum, 1984; Parker & Schneider, 1974; Schneider, Parker, Farrell, & Kanow, 1976). In agreement with this prediction, in an unpublished study on brightness bisection, I have accordingly found that the subjects who failed to pass Gage's test were a minority (21%) of the subjects who passed Pfanzagl's test.

Richardson (1929) proposed to measure sensory attributes by the cross-modality matching method currently known as graphic rating, in which subjects match the apparent length of a selected part of a line to the magnitude of a target sensory attribute. The physical length of the part of the rating line selected by the subject is interpreted as the measure of the target sensory magnitude. This interpretation is based on the implicit, but unproven, assumption that the apparent length of the selected part of the rating line is linearly related to the corresponding physical length. (Since the rating line is typically short, and thus involves a small part of the psychophysical function, physical length is in this case an admissible approximation to the corresponding sensory measure.) This unproven assumption has been restated more recently in the proposal of using apparent length as a reference attribute and measure magnitudes of other sensory attributes by matching them cross-modally to apparent length (Krantz, 1972). The results of the present study provide a direct verification of this assumption by confirming that, essentially, $n = k = m = l$. 268
REFERENCES


BOUNDARY EXTENSION AND MEMORY FOR AREA AND DISTANCE

Jon R. Courtney and Timothy L. Hubbard
Department of Psychology, Texas Christian University, Fort Worth, Texas 76129 USA
jonr Courtney@gmail.com; t hubbard@tcu.edu

Abstract

Memory for a previously viewed picture of a scene often includes details that might have been present just beyond the boundaries of that picture. This is known as boundary extension (Intraub & Richardson, 1989) and has been proposed to reflect the anticipatory nature of representation (Intraub, 2002). Another possible explanation of boundary extension involves changes in remembered distance or size (Hubbard, 1996). To examine whether boundary extension is due to changes in remembered distance, participants judged distances to objects in 3-D scenes. Results were consistent with previous research in memory psychophysics. To examine whether boundary extension is due to changes in remembered size, participants recalled boundaries of scenes while object size remained unchanged. Results were consistent with previous research in boundary extension. The data suggest boundary extension is not due to changes in memory for distance or size.

When observers view a picture of a scene, their subsequent memory for that scene often includes details that were not present in the picture, but that might have been present just outside the boundaries of the viewed scene (see Figure 1). This is referred to as boundary extension (Intraub & Richardson, 1989), and it has been suggested to reflect the